A Symbol for Continued Proportionality

14 May 2021

Virtuellen Tag zur Mathematikgeschichte Hildesheim

Staffan Rodhe
Uppsala, Sweden

staffan.rodhe@gmail.com

What is Continued Proportionality?

Geometric

A, B and C are in a continued proportion:

A.B :: B.C

$$\frac{A}{B} = \frac{B}{C}$$

$$AC = B^2$$

A, B, C, D, … are in a continued proportion:

A.B :: B.C :: C .D :: …

$$\frac{A}{B} = \frac{B}{C} = \frac{C}{D} = \cdots$$

Arithmetic

A, B, C, D, … are in continued proportion:

$$B - A = C - B = D - C = \cdots$$

When of several Quantities the $\left\{\begin{matrix}\text{Difference}\\ \text{Quotient}\end{matrix}\right\}$ *of the* 1st. *and* 2d. *is the same with that of the* 2d. *and* 3d. *they are said to be in a* Continued $\left\{\begin{matrix}\text{Arithmetic}\\ \text{Geometric}\end{matrix}\right\}$ **Proportion.**

Thus, $\left\{\begin{matrix} a, a+d, a+2d, a+3d, a+4d, \\ a, a-d, a-2d, a-3d, a-4d, \end{matrix}\right\}$ &c. is a Series of *Continued Arithmetic Proportionals*, whose *Common Difference* is d.

And $\left\{\begin{matrix} a, & ar, & arr, & arrr, & arrrr, & ar^5 \\ a, & \frac{a}{r}, & \frac{a}{rr}, & \frac{a}{rrr}, & \frac{a}{rrrr}, & \frac{a}{r^5} \end{matrix}\right\}$ &c.

is a Series of *Continued Geometric Proportionals*, whose *Common Multiplier* is $\left\{\begin{matrix} \frac{r}{1} \\ \frac{1}{r} \end{matrix}\right\}$ or whose Ratio is that of $\left\{\begin{matrix} 1 \text{ to } r \\ r \text{ to } 1 \end{matrix}\right\}$.

Synopsis Palmariorum Matheseos:

OR, A

New Introduction

TO THE

MATHEMATICS:

Containing the

PRINCIPLES

OF

Arithmetic & *Geometry*

DEMONSTRATED,

In a Short and Easie Method;

WITH

Their Application to the most Useful Parts thereof: As, Resolving of *Equations*, *Infinite Series*, Making the *Logarithms*; *Interest*, *Simple* and *Compound*; The Chief Properties of the *Conic Sections*; Mensuration of *Surfaces* and *Solids*; The Fundamental Precepts of *Perspective*; *Trigonometry*; The *Laws* of *Motion* apply'd to *Mechanic Powers*, *Gunnery*, &c.

Design'd for the Benefit, and adapted to the Capacities of BEGINNERS.

By *W. JONES.*

LONDON: Printed by *J. Matthews* for *Jeff. Wale* at the *Angel* in St. *Paul's* Church-Yard, 1706.

Therefore, in any *Geometric Proportion*, when the *Antecedent* is $\left\{\begin{matrix}\textit{less}\\ \textit{greater}\end{matrix}\right\}$ than the *Consequent*, the *Terms* may be expreſs'd by $\left\{\begin{matrix} a \text{ and } ar \\ a \text{ and } \frac{a}{r}\end{matrix}\right\}$.

Definition III.

THose *Quantities whoſe* Exceſs *or* Quotients *are the ſame, are call'd* Proportionals.

Case I.

When of ſeveral Quantities the $\left\{\begin{matrix}\text{Difference}\\ \text{Quotient}\end{matrix}\right\}$ *of the* 1ſt. *and* 2d. *is the ſame with that of the* 2d. *and* 3d. *they are ſaid to be in a* Continued $\left\{\begin{matrix}\text{Arithmetic}\\ \text{Geometric}\end{matrix}\right\}$ Proportion.

Thus, $\left\{\begin{matrix} a, & a+d, & a+2d, & a+3d, & a+4d, \\ a, & a-d, & a-2d, & a-3d, & a-4d, \end{matrix}\right\}$ &c. is a Series of *Continued Arithmetic Proportionals*, whoſe *Common Difference* is d.

And $\left\{\begin{matrix} a, & ar, & arr, & arrr, & arrrr, & ar^5 \\ a, & \frac{a}{r}, & \frac{a}{rr}, & \frac{a}{rrr}, & \frac{a}{rrrr}, & \frac{a}{r^5} \end{matrix}\right\}$ &c. is a Series of *Continued Geometric Proportionals*, whoſe *Common Multiplier* is $\left\{\begin{matrix} \frac{r}{1} \\ \frac{1}{r} \end{matrix}\right\}$ or whoſe Ratio is that of $\left\{\begin{matrix} 1 \text{ to } r \\ r \text{ to } 1 \end{matrix}\right\}$.

Note, That the Sign $\div$ Signifies *Continued Proportion*.

∺

The Explication of the Signs and Characters used in this Treatise.

Sign		Signifies
$=$	Signifies	*Equality*, or equal to.
$>$		*Majority*, or greater than.
$<$		*Minority*, or less than.
$+$		*More*, or to be added.
$-$		*Less*, or to be subtracted.
$\backsim$		The *Difference* or *Excess*.
$\times$		*Multiplied* by.
$\surd$		*Radicality*.
$\div\!\div$		*Continual Proportion*.

Other Signs or Abbreviations of Words that occur, are explain'd in their own Places.

And in three continual Proportionals, *The square of the Middle divided by one of the Extremes, gives the other*: For if A . B . C $\because$, (that is, A . B :: B . C, and therefore BB = AC;) then (dividing both by A,) $\frac{BB}{A} = C$. Or (both by C) $\frac{BB}{C} = A$.

These therefore will be in continual Proportion.

$$a . b . \frac{bb}{a} . \frac{bbb}{aa} . \frac{b^4}{a^3} . \frac{b^5}{b^4} . \&c. \because .$$

Mr. William Oughtred's
KEY
OF THE
Mathematicks.

Newly Tranſlated from the Beſt EDITION

VVith NOTES,

Rendring it Eaſie and Intelligble to leſs Skilful *Readers.*

R In which alſo, Kenyon

Some PROBLEMS

Left Unanſwer'd by the Author are Reſolv'd.

Abſolutely neceſſary

For all *Gagers, Surveyors, Gunners, Military-Officers, Mariners,* &c.

Recommended by Mr. E. HALLEY, *Fellow of the* Royal Society.

LONDON:

Printed for John Salusbury, at the *Riſing-Sun* in *Cornhil.* MDCXCIV.

it requires the fourth terme to be : the leſſer it is, the greater.

8. Continued proportion $\div$, is when all the middle terms between the firſt and the laſt, are both conſequents and antecedents of Rations : as 8, 12, 18, 27, are $\div$ For $8.12::12.18::18.27$.

Alſo $\alpha, \beta, \frac{\beta q}{\alpha}\ \frac{\beta c}{\alpha q},\ \frac{\beta qq}{\alpha c},\ \frac{\beta qc}{\alpha qq}$ &c. are $\div$

Wherefore if in this *ſeries*, the laſt terme be ω, and the ſumme of all the terms of the whole progreſſion be Z : then $Z-\omega$ will be the ſumme of all the antecedents; and $Z-\alpha$, the ſum of all the conſequents.

9. If foure magnitudes be proportionall, $A.\alpha::B.\beta$. they ſhall alſo be proportionall being Alterned, and Inverted, and Compounded, and Divided, and Converted, and in Mixture.

$A\alpha::B.\beta$.

Altern. $A.B::\alpha.\beta$.
Invert. $\alpha.A::\beta.B$.
Compound. $A+\alpha.\alpha::B+\beta.\beta$.
or. $A+B.B::\alpha+\beta.\beta$.
Divid. $A-\alpha.\alpha::B-\beta.\beta$.
or. $A-B.B::\alpha-\beta.\beta$.
Convert. $A.A\pm\alpha::B.B\pm\beta$.
or. $A.A\pm B::\alpha.\alpha\pm\beta$.
In Mixture. $A+\alpha.A-\alpha::B+\beta.B-\beta$.
or. $A+B.A-B::\alpha+\beta.\alpha-\beta$.

10. If never ſo many magnitudes be proportionall it ſhall be as one of the antecedents is to its conſequent; ſo the ſumme of the antecedents to the ſumme of the conſequents. If it be $A.\alpha::B,\beta::C.\gamma::D,\delta$: Then will $A.\alpha::A+B+C+D.\alpha+\beta+\gamma+\delta$.

$$\text{For}\begin{cases} A.\alpha::B.\beta, \text{ and compounded.} \\ A+B.\alpha+\beta::(B.\beta)C.\gamma. \text{ And} \\ A+A+C.\alpha+\beta+\gamma::(C.\gamma)D.\delta.\&c. \end{cases}$$

Alſo in continuall proportionalls, $\alpha.\beta::Z-\omega.Z-\alpha$. Wherefore $\alpha Z-\alpha q=\beta Z-\beta\omega$, or $\beta Z-\alpha Z=\beta\omega-\alpha q$. Hence by the way we may ſee how to finde the ſumme of all the termes $\div$, or of progreſſion Geometricall, by this rule $\left\{\frac{\beta\omega-\alpha q}{\beta-\alpha}=Z.\right.$

11. If the antecedents of many proportions bee equall; then as one of the antecedents is to the ſumme of his conſequents; ſo will the other antecedent bee to the ſumme of his. Let it be $A.B::\alpha.\beta$. and $A.C::\alpha.\gamma$. and $A.D::\alpha.\delta$. Then $A.B+C+D::\alpha.\beta+\gamma+\delta$. it appeareth out of the former demonſtration, the terms being alternly placed.

12. If the conſequents of two Rations be equall, they are as their Antecedents : But if the Antecedents be equall, they are reciprocally as their conſequents.

$\frac{7}{1}.\frac{9}{1}::7.9$. And $\frac{1}{9}.\frac{1}{7}::7.9$.

13. If twice foure magnitudes be alike proportionall; then both their ſummes and alſo their differences ſhall be proportionall.

14. If foure proportionall magnitudes be multiplied or divided by foure other proportionall magnitudes; their products or quotients ſhall alſo be proportional.

15. The

Erste höchst seltene Ausgabe der Clavis von Oughtred.

Die Logarithmen-Tafeln von Faulhaber sind immer noch brauchbar und selten.

ARITHMETICÆ IN NVMERIS ET SPECIEBVS INSTITVTIO: QVÆ TVM LOGISTICÆ, TVM ANALYTICÆ, ATQVE ADEO TOTIVS MATHEMATICÆ, QVASI CLAVIS EST.

AD NOBILISSIMVM SPEctatissimumque iuvenem D N. GVILELMVM HOVVARD, Ordinis, qui dicitur, Balnei Equitem, honoratissimi DN. THOMÆ, Comitis ARVNDELIÆ & SVRRIÆ, Comitis MARESCHALLI ANGLIÆ, &c. filium.

[illegible] tenet

LONDINI,
Apud THOMAM HARPERVM.
M.DC.XXXI.

Constat 1.4.

rorum autem ad se inuicem habitudo inuenitur diuidendo antecedentem per consequentem, vt 31 ad 7 ratio est $4\frac{3}{7}$, hoc est quadrupla supertripartiens septimas. Sic etiam in partibus ratio $\frac{3}{2}$ ad $\frac{2}{3}$ est $\frac{9}{4}$, vel 9 ad 4, dupla & sesqui quarta.

2 Quare si numerus duos numeros multiplicet; facti erunt multiplicatis proportionales. Et si numerus duos numeros diuidat; quoti erunt diuisis proportionales.

$$4 \times \begin{cases} 7. & 28 \\ 9. & 36 \end{cases} \qquad A \times \begin{cases} B. & BA \\ C. & CA \end{cases}$$

3 Si quatuor numeri sint proportionales, factus ab extremis æquatur facto a medijs. vt in his quia $7.\ 9 :: 28.\ 36$. erit $7 \times 36 = 9 \times 28$. & quia $R.\ S :: Z.\ A$, erit $RA = ZS$.

4 Hinc sequitur aurea (quæ dicitur) regula proportionis: Si tribus numeris datis; rectangulum sub secundo & tertio adplicetur ad primum: hoc est, si secundus multiplicet tertium, & primus diuidat factum: quotus erit tribus datis quartus proportionalis. vt

$$7.\ 9 :: 28.\ 36. \qquad 5.\ 12 :: 8.\ 19\tfrac{1}{5}$$

$$7)\ \underline{9}\,(36 \qquad 5)\ \underline{2}\,(19\tfrac{1}{5}$$

$$252 \qquad\qquad 96$$

item $R.\ S :: Z.\ \frac{ZS}{R}$: Et quia modo positum fuit $R.\ S :: Z.\ A$: erit $\frac{ZS}{R} = A$.

5 E tribus numeris datis ad quartum proportionalem inueniendum; terminus is, per quem fit quæstio, in proportione directa est secundus, vel tertius: at

at in proportione reciproca semper est primus.

6 Directa quidem proportio est, quando terminus is, per quem fit quæstio, quò maior est, eò quartum maiorem requirit: & quò minor, eò minorem.

7 Reciproca proportio est, quando terminus is per quem fit quæstio, quò maior est eò quartum minorem requirit: & quò minor, eò maiorem.

8 Proportio continua $\div\!\!\div$ est, quando termini omnes medij inter primum & vltimum, rationum sunt tum consequentes, cum antecedentes. vt 8. 12. 18. 27. Item

$$\alpha.\ \beta.\quad \frac{\beta q}{\alpha}.\ \frac{\beta c}{\alpha q}.\ \frac{\beta qq}{\alpha c}.\ \frac{\beta qc}{\alpha qq}.\quad \&c.$$

9 Si quatuor numeri sint proportionales $A.\ \alpha :: B.\ \beta$: etiam alternè, & inuersè, & compositè, & diuisim & conuersè proportionales erunt

	alternè,	$A.\ B :: \alpha.\ \beta.$
	inuersè,	$\alpha.\ A :: \beta.\ B.$
	compositè,	$A+\alpha.\ \alpha :: B+\beta.\ \beta.$
	diuisim,	$A-\alpha.\ \alpha :: B-\beta.\ \beta.$
	conuersè,	$A.\ A+\alpha :: B.\ B+\beta.$

10 Si quotlibet numeri sint proportionales: erit vt vnus antecedens ad suum consequentem, sic summa antecedentium, ad summam consequentium. Esto $A.\ \alpha :: B.\ \beta :: C.\ \gamma$: erit $A.\ \alpha :: A+B+C.\ \alpha+\beta+\gamma$.

11 Si plurium proportionum antecedentes sint æquales: erit vt vnus antecedens ad summam suorum consequentium, sic alter antecedens ad summam suorum. Esto $A.\ B :: \alpha.\ \beta$: & $A.\ C :: \alpha.\ \gamma$: Erit $A.\ B+C :: \alpha.\ \beta+\gamma$.

12 Si

mæ ⁒ tum addendum est tum auferendum.

7 Si linea vtcunque secta multiplicetur in alterutrum suum segmentum: rectangulum illud duplicatum, plus quadrato alterius segmenti, æquale erit quadratis totius & segmenti multiplicantis. hoc est, Si $Z = A + E$: erit $2ZA + Eq = Zq + Aq$. Et $2ZE + Aq = Zq + Eq$.

8 Si linea vtcunque secta multiplicetur in alterutrum suum segmentum: rectangulum illud quadruplicatum, plus quadrato alterius segmenti, æquale erit quadrato lineæ totius auctæ segmento multiplicante. hoc est, Si $Z = A + E$: erit $Q{:}Z + A{:} = 4ZA + Eq$. Et $Q{:}Z + E{:} = 4ZE + Aq$.

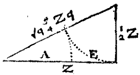
9 Si linea bisecetur & secus: quadrata segmentorum inæqualium simul duplicia sunt quadratorum bisegmenti & intersegmenti. hoc est, Si $Z = A + E$ erit $Aq + Eq = \frac{2}{4}Zq + \frac{2}{4}Xq$.

10 Si linea bisecta augeatur: quadrata totius auctæ & augmenti, simul duplicia sunt quadratorum bisegmenti, & bisegmenti aucti. hoc est, Si $Z = A + E$: erit $Zq + Eq = \frac{2}{4}Aq + 2Q\,\frac{1}{2}A + E$.

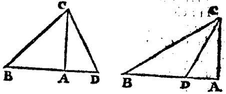
11 Datam rectam lineam ita secare, vt rectangulum sub tota & minore segmento æquale sit quadrato maioris. hoc modo, Si $Z = A + E$, sitque $\sqrt{q}\,\frac{5}{4}Zq - \frac{1}{2}Z = A$: erit $ZE = Aq$. Nam $\frac{5}{4}Zq = Q{:}\frac{1}{2}Z + A{:} = \frac{1}{4}Zq + ZA + Aq$. Quare $ZA + Aq = Zq = ZA + ZE$. Ergo.

Atque

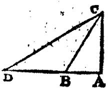
Atque hinc patet modus secandi datam lineam Z secundum mediam & extremam rationem: hoc est vt sint Z. A. E ∺



12 In triangulo BCD, si ang. B sit acutus: $BCq + BDq = DCq + 2BD \times BA$. Nam $BCq - BAq = CAq = DCq - Q{:}BD - BA$
47 e 1 $= DCq - BDq + 2BD \times BA - BAq$.



13 In triangulo BCD, si ang. B sit obtusus: $BCq + BDq = DCq - 2BD \times BA$. Nam $BCq - BAq = CAq$
47 e 1 $= DCq - Q{:}BD + BA$
$- BDq - 2BD \times BA - BAq$



 14 Dato

12. Probl. Three points being given, not lying direct, to draw a circumference through them. 25 e 3.

13. Probl. The Base and Cathetus of a rectangled Triangle being given, to finde the Hypotenuse: or to adde a Quadrat to a Quadrat.

14. Probl. The Hypotenuse and Base of a rectangled Triangle being given, to finde the Cathetus; or to take a Quadrat out of a Quadrat.

15. Probl. To finde the Ration of Two like Figures; seeke a Third Proportional.

16. Probl. A Figure being given, to frame a like Figure, in a Ration given. Seeke a Meane-proportional betweene the Side thereof, and a like Side.

17. Probl. In a circle given to inscribe an ordinate or regular Hexagon. 15 e 4.

18. Probl. In a circle given to inscribe an ordinate Decagon. Cut the Semidiamiter of the Circle after the Extreme and Meane Ration, by 11 e 2.

19. Probl. In a circle given to inscribe an ordinate Pentagon. Seek the Hypotenuse of a rectangled Triangle, whose Base is the Side of an Hexagon, and Cathetus the Side of a Decagon.

Chap.

Chap. XIX.

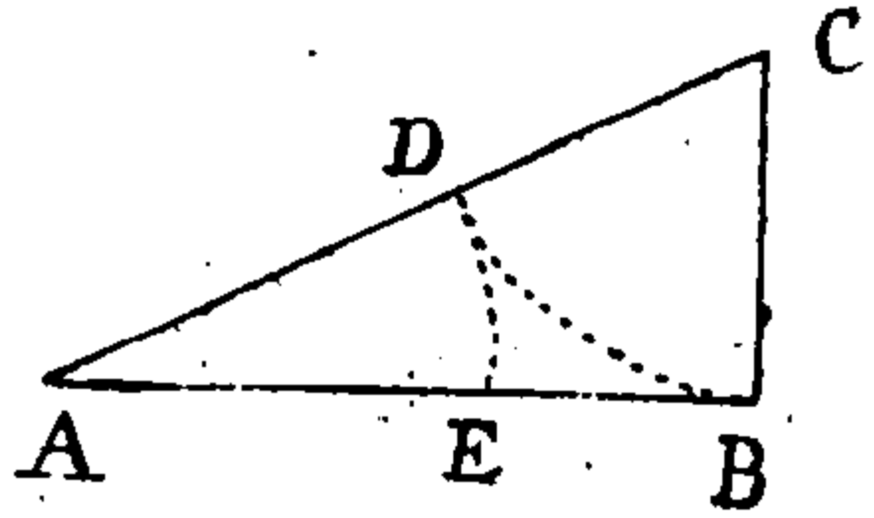
Examples of Analytical Æquation, for inventing of Theoremes, and resolving of Problemes. At which marke (as it were) the Precepts hitherto delivered, do principally aime.

Probl. I. THE Invention of 11 e 2. Namely, to cut B a Right line given, so that the Rectangle under the whole B, and the lesser Segment, may be equall to the Quadrat of the greater Segment.

Let the greater Segment be put A: the lesser shall be B—A. Draw B—A into B; and there shall bee made $Bq-BA=Aq$: or $Aq+BA=Bq$. Wherefore $\sqrt{u}:Bq+\frac{1}{4}Bq:-\frac{1}{2}B=A$; by 9 Ch. XVI.

Which Theoreme is expressed by words thus: If to the Quadrat of the Line given, be added a quarter of the said Quadrat; and from the Quadrat-side of the Summe, be taken Half the line given; the Remainder shall be the greater Segment.

Now it is Geometrically effected thus; Draw $AB=B$: And to it at Right angles, set $BC=\frac{1}{2}B$; and draw the Hypotenuse AC: It shall be $AC=\sqrt{u}:Bq+\frac{1}{4}Bq$. Cut off $CD=BC$; And the Remainder shal be $AD=\sqrt{u}:Bq+\frac{1}{4}Bq:-\frac{1}{2}B$.



Lastly, measure $AE=AD$, for the greater Segment.

G Probl.

6.

Geometrica et Algebraica.

(192a, b.)

1–187. 191–192. 199–404. Mellan 220–221: En tryckt tabell.
Efter 404: En liten lapp med [illegible]

1870 c/a. J. A. [illegible]

Analogia loco aequationum

Statui etiam potest analogia statim loco aequationum; ut si linea quaedam quaeratur, pariter altera, tunc illae comparari possunt cum signo $\div$, ut si una linea sit $\frac{ab}{m}+f$ et altera linea $\frac{am}{d}+n$ tunc ponantur simul cum signo $\div$ sit $\frac{ab}{m}+f \div \frac{am}{d}+n$.

Si linea ignota sit intermixta, etiam potest illa assumi modo habeat quantitas lineae vel rei quaesitae, ut $\frac{abx}{m}$ alii lineae $\frac{ab}{d}$ sit $\frac{abx}{m} \div \frac{ab}{d}$

Cum his autem operari potes eodem modo ut cum aequationibus, sed tantum per multiplicationem et divisionem, usque tamen semper remanet eadem analogia. pariter si extrahantur utrinque radices vel si ad dimensionem altiorem eleventur: ~~sed non per~~ ad

Si desideretur analogia arithmetica, sit haec per additionem et subtractionem, si addatur vel subtrahatur, remanet tamen semper analogia arithmetica, non geometrica.

Analogiae loco aequationum
Quaeri etiam potest analogia statim loco aequationis; ut si linea quaedam quaeratur, pariter altera, tunc illae componi possunt cum signo ∺, ut si una linea sit $\frac{ab}{m} + f$ et altera linea $\frac{am}{d} + n$ tunc ponantur simul cum signo ∺ fit $\frac{ab}{m} + f$ ∺ $\frac{am}{d} + n$.

Translation: Proportions instead of equations
Instead of an equation a proportion can be immediately searched for; e.g. if a line is searched for and equally another, e.g. if one line is $\frac{ab}{m} + f$ and the other line $\frac{am}{d} + n$ then both can be taken with the symbol ∺ and it becomes $\frac{ab}{m} + f$ ∺ $\frac{am}{d} + n$.

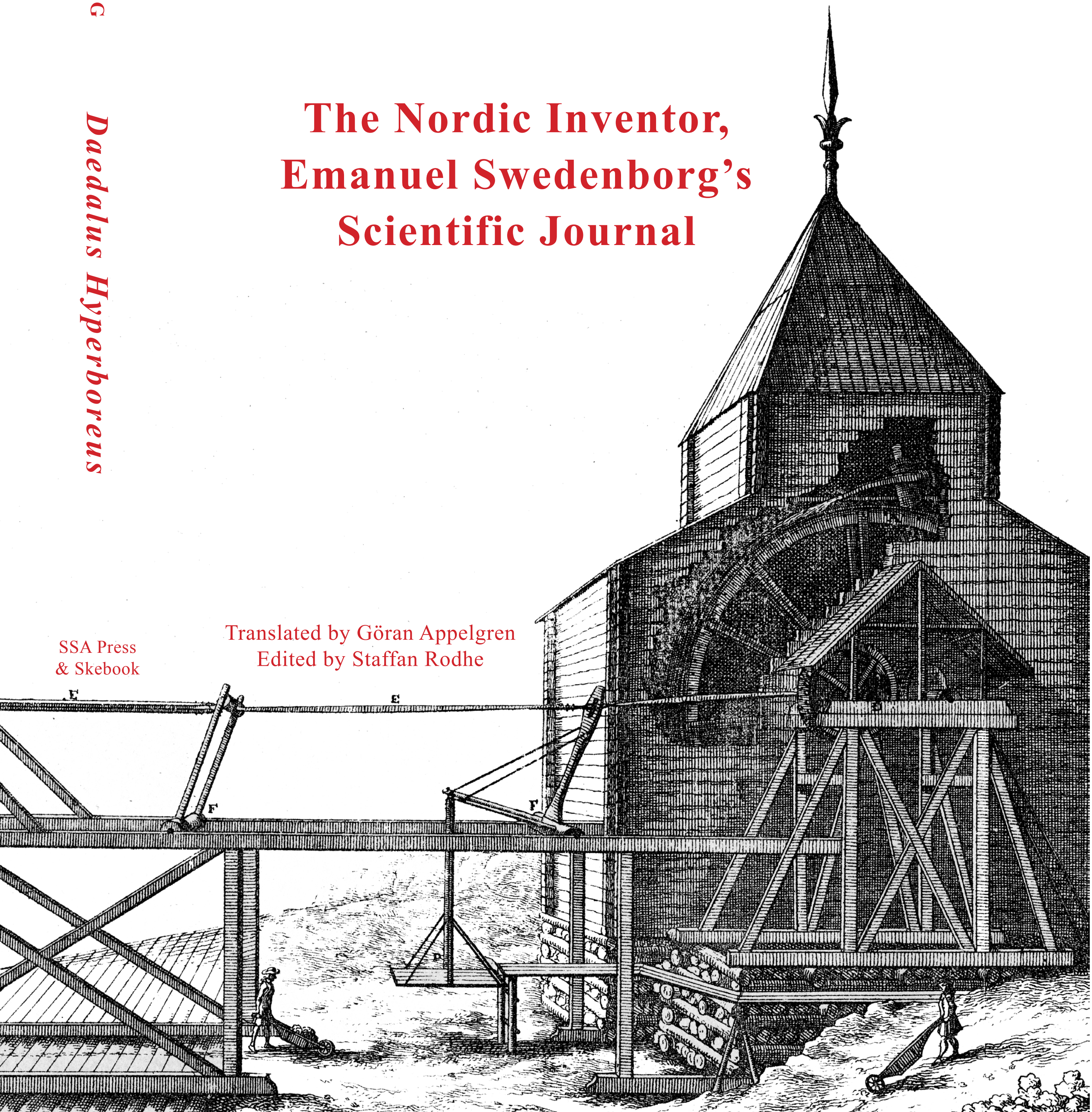
What does this mean?

SWEDENBORG

Daedalus Hyperboreus

Daedalus Hyperboreus

The Nordic Inventor, Emanuel Swedenborg's Scientific Journal



SSA Press
& Skebook

Translated by Göran Appelgren
Edited by Staffan Rodhe

3. The method is thus: when one has two lines, areas, or bodies and one wishes a proportionality between them, then one takes the algebraic expressions of both and sets them against one another with a so called “signo analogico $\div\!\!\!\!\cdot\!\cdot$ and operates with only divisions and multiplications in the same manner as with equations, until one has reduced both numbers to the smallest fraction. And since multiplication and division do not at all change any geometric analogy it follows thereof that one can see the proportionality of the two lines or bodies in the smallest number.

4. To find what ratio a cube has to an enclosed cylinder, one sets (d) = diameter or the side of the cube, (c) = circumference, and then the volume of the cube is found to be = ddd, and that of the enclosed cylinder = $\frac{\mathrm{ddc}}{4}$. Thus one sets ddd $\div\!\!\!\!\cdot\!\cdot$ $\frac{\mathrm{ddc}}{4}$ and uses division and multiplication to obtain a smaller fraction thereof, namely 4d $\div\!\!\!\!\cdot\!\cdot$ c; that is in numbers $\frac{14}{11}$ or 14 $\div\!\!\!\!\cdot\!\cdot$ 11 or as four times the diameter relates to the circumference, as the volume of the cube relates to that of the cylinder. And one sees thereof that an oblong also has the same ratio to an enclosed long cylinder, as also a square surface to the area of an enclosed circle, namely as 14 to 11.

A cube to an enclosed sphere: The cube is as before ddd, and the enclosed sphere which is of the same diameter as the side of the cube = $\frac{\text{ddc}}{6}$.

The analogy is ddd $\doteqdot \frac{\text{ddc}}{6}$ and by means of multiplication and division this becomes 6d $\doteqdot$ c, that is 6 diameters is related to the periphery as a cube is related to an enclosed sphere, or in numbers as 21 to 11, as before. This is the lowest even number that the volume or weight of a cube has to the volume or weight of an enclosed sphere, which touches all four sides of the cube with its surface. The same ratio is also found in an oblong volume or weight against that of an enclosed oval, as also if many globes or balls were to be stacked in a hexahedron (cube), and closely around them would be made a square-formed house, then the volume of the house to that of the balls together would be as $\frac{21}{11}$.

That is 6d.c :: ddd. $\frac{\text{ddc}}{6}$ :: 21.11

What is this about continued proportionality?

6. I wish to know the proportion of a parabola, or the common characteristics of its ordinates, parameter, and axes. Its equation thereof is $px = yy$ or $x = \frac{yy}{p}$. Thus if the analogy is set as $y \doteqdot yy/p$, it is found to be like $p \doteqdot y$. That is p, y and x are in a continued geometric proportion.

$$x \doteqdot y$$

$$y \doteqdot p$$

$$x.y :: y.p$$

In Swedenborg's book on algebra, *Regelkonsten,* which he wrote later in 1717, he shows six examples of how he uses his analogy method but with the simpler sign :: . Interestingly enough he writes more about it:

> You can as well in everything else put two expressions against each other to see how one relates the other. The reason for this is that if you multiply or divide two proportional expressions with the same then the proportion between the two new expressions is the same but expressed with smaller fractions. This is something that has been performed in Daedalus vol. V, and it is a new way to do which has not been used before, and it has its benefits here and there.