

# Online Symposium on PEG SOLITAIRE ON GRAPHS

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For several centuries, the single-player board game (Peg) Solitaire fascinated (and still fascinates) people around the globe and, eventually, the mathematical research community. Only recently (in 2011), but certainly very naturally, peg solitaire has been extended to graphs. Since then, this game variant has attracted a growing number of researchers.

The rules are simple: For an (undirected) graph  $G = (V, E)$ , we put pegs in some of the vertices of  $G$ . Given three vertices  $u, v, w$  with pegs in  $u$  and  $v$  and a hole in  $w$  such that  $uv, vw \in E$ , we can jump with the peg from  $u$  over  $v$  into  $w$ , removing the peg in  $v$ :



Several fascinating and challenging problems occur when considering peg solitaire on graphs. How many pegs can we remove at most from a graph? How many at least? Determine, given some graph, the minimum number of additional edges needed until it is possible to remove all but one peg. Many more interesting problems exist and most of them are open or only partially solved. Additionally, several intriguing variants of the game were introduced in the last couple of years, each of which coming with its own family of exciting research questions.

The goal of this symposium is to connect people who are active or interested in this field.

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The symposium will be held online on

June 20th/21st of 2022

To access the symposium, please use the following link:

<https://fu-berlin.webex.com/fu-berlin/j.php?MTID=m40061688b6e499f1ad226337b9d6391f>

You can either install the Webex-Client and join via the client or you can join directly with your browser. If you have any questions please feel free to write to Martin Kreh (see contact details below).

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The schedule and abstracts of the talks can be found on the next pages. All times are UTC (beginning in the evening of June 20th and ending in the morning of June 21st). Please account for different time zones, some of which are:

Time zone	HST	CDT	EDT	CEST	WIB	JST
UTC $\pm$	-10	-5	-4	+2	+7	+9

Anyone wishing to participate is very welcome.

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Organisers:

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Martin Kreh (Universität Hildesheim), [kreh@imai.uni-hildesheim.de](mailto:kreh@imai.uni-hildesheim.de)

# Schedule

Time	Activity
23:15 – 23:25	Opening/Get together
23:25 – 23:45	Martin Kreh - Pagoda functions for peg solitaire on graphs
23:45 – 00:15	Valentino Vito - Peg solitaire and Conway's soldiers on infinite graphs
00:15 – 00:25	Short break
00:25 – 00:45	Aaron D. Gray - Peg Solitaire Variations: Double Jump and Duotaire
00:45 – 01:05	Jan-Hendrik de Wiljes - Path-Stick Solitaire
01:05 – 01:15	Short break
01:15 – 01:45	Robert A. Beeler - Peg Solitaire on Graphs: A Retrospective with Open Problems
01:45 – open end	Discussion/Future research cooperation

## Abstracts

**Martin Kreh - Pagoda functions for peg solitaire on graphs** In this talk, we show a new approach for considering peg solitaire on graphs, i.e., using pagoda functions. Pagoda functions are a classical tool for the original peg solitaire, but are, until now, almost never used for peg solitaire on graphs. We will show some properties that pagoda functions on graphs need to satisfy. We also present an algorithm that constructs a pagoda function for an arbitrary graph (as long as this graph has a vertex of degree 1) and show how we can bound the peg solitaire number using this function.

**Valentino Vito - Peg solitaire and Conway's soldiers on infinite graphs** Reminiscent of peg solitaire, Conway's soldiers is a game played on  $\mathbb{Z}^2$  with pegs initially located on the half-space  $y \leq 0$ . The goal is to bring a peg as far as possible on the board using peg solitaire jumps. Conway famously showed that bringing a peg to the line  $y = 5$  is impossible with finitely many jumps. In this talk, we introduce an analogous impossibility property on graphs. In addition, we generalize peg solitaire on finite graphs to an infinite game played on countable graphs. We also consider problems involving Cartesian products of graphs in this setting.

**Aaron D. Gray - Peg Solitaire Variations: Double Jump and Duotaire** In this talk, we explore two variations of peg solitaire on graphs. In double jump peg solitaire, each peg requires two (not necessarily consecutive) jumps to be eliminated from the graph. Peg duotaire is a two player variation in which players jump pegs on alternate turns. This talk serves as an introduction to both variations with illustrations and an overview of major results and open problems.

**Jan-Hendrik de Wiljes - Path-Stick Solitaire** Instead of playing with pegs on the vertices of a graph, one might place sticks onto the graphs edges. Defining the jump possibilities analogously to peg solitaire yields the new game path-stick solitaire. This game will be considered on instances of several (well-known) graph classes in this talk and some analogues to peg solitaire results will be presented. Naturally, many open problems appear when studying path-stick solitaire, some of which will be discussed at the end of the talk.

**Robert A. Beeler - Peg Solitaire on Graphs: A Retrospective with Open Problems** In 2011, the traditional game of peg solitaire was generalized to graphs. In this talk, I will give a survey focusing on the contributions I have made with my co-authors to this area. In addition to the usual goal of minimizing the number of pegs at the end of the game, we will be discussing the fool's solitaire problem (leave as many pegs as possible, assuming the player jumps whenever possible), and the recently introduced variation where jumps and merges are allowed. Throughout the talk, there will be a number of open problems and conjectures.

